Module III: Curves and Surfaces

Computer Aided Design & Analysis

1. Representation of Curves

Curves in CAD are essential for creating complex shapes and forms. They can be represented in several ways:

- **Explicit Form**: $y = f(x)$ (limited flexibility)
- **Implicit Form**: $F(x, y) = 0$ (for circles, ellipses, etc.)
- **Parametric Form**: Most versatile, uses a parameter (t) so that $x = x(t)$, $y = y(t)$. Ideal for CAD since any curve shape can be described and manipulated efficiently.

2. Key Parametric Curves in CAD

a) Hermite Curves

- Defined by: Two endpoints and tangent vectors at each.
- Useful for: Controlled transitions, animation paths, and local modification.
- Equation:
  $C(t) = h_1(t)P_0 + h_2(t)P_1 + h_3(t)T_0 + h_4(t)T_1$
  where P_0, P_1 are endpoints, T_0, T_1 are tangents, and $h_i(t)$ are basis functions.

b) Bézier Curves

- Defined by: A set of $n+1$ control points ($P_0, \dots, P_n$).
- Equation:
  $B(t) = \sum_{i=0}^n {n \choose i} (1-t)^{n-i} t^i P_i$, $t \in$
- Key Properties:
  - Curve starts at P_0 and ends at P_n.
  - Entire curve lies within the convex hull of the control points.
  - Widely used in graphic design, CAD, and animation.

c) B-spline Curves

- Generalization of Bézier curves enabling local shape control.
- Defined by: Control points, degree ($ p $), and knot vector (partition of the parameter domain).
- Equation:
  $ C(t) = \sum_{i=0}^{n} N_{i,p}(t) P_i $
  where $ N_{i,p}(t) $ are B-spline basis functions.
- Properties:
  - Modifying a control point affects only a segment (locality).
  - Versatile degree and smoothness adjustment.

d) Rational Curves (NURBS)

- **Non-Uniform Rational B-Splines (NURBS):** Most general form, includes weights for exact representation of conic sections.
- Equation:
  $ C(t) = \dfrac{\sum_{i=0}^{n} N_{i,p}(t) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(t) w_i} $
- Used for: Circles, ellipses, and flexible freeform curves in modern CAD.

3. Surface Modelling

Surface modeling in CAD allows creation of complex 3D objects using mathematical and geometric techniques.

a) Parametric Representation

Surfaces are often defined as:
$ S(u, v) = [x(u, v),\ y(u, v),\ z(u, v)] $
where $ (u, v) $ are parameters.

b) Planar Surface

- Simplest surface type, defined by three or more coplanar points.
- Parametric form:
  $ S(u, v) = P_0 + u(P_1 - P_0) + v(P_2 - P_0) $
  (for triangle, $ u, v \geq 0, u + v \leq 1 $).

c) Surface of Revolution

- Generated by rotating a planar curve about an axis.
- Parametric form (for rotating profile $ r(z) $ around z-axis):
  $ x = r(z) \cos\theta,\quad y = r(z) \sin\theta,\quad z = z $
  where $ \theta \in [0, 2\pi] $.

d) Coons and Bicubic Patches

Coons Patch:

- Used for interpolating four boundary curves.
- Blends boundary conditions smoothly to fill the patch.

Bicubic Patch:

- Parametric surface defined by cubic polynomials in both u and v.
- Equation:
  $S(u, v) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} u^i v^j$
- Capable of modeling smooth, organic freeform surfaces.

e) Bézier and B-spline Surfaces

Bézier Surface:

- Defined by a grid of control points.
- Parametric form:
  $S(u, v) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_{i,m}(u) B_{j,n}(v) P_{ij}$
  where $B_{i,m}(u)$, $B_{j,n}(v)$ are Bézier basis polynomials.

B-spline Surface:

- Uses tensor product of B-spline basis functions in both parameters u and v.
- Provides local modification (changing one control point affects only a small part of the surface).
- Foundation of NURBS surfaces—standard in industrial CAD.

4. Overview Table: Common Curves and Surfaces

Entity	Key Features	Application Examples
Hermite Curve	Endpoints and tangent control	Animation paths, splines
Bézier Curve	Control points, convex hull property	Font design, CAD sketches
B-spline Curve	Local shape control, flexible degree	Automotive, ship hulls
NURBS Curve/Surf.	Weights & knots for complex, exact shapes	Aerospace, industrial CAD
Planar Surface	Simple, flat geometry	Plates, base features
Surface of Revolution	Axisymmetry, efficient to create	Bottles, nozzles
Coons Patch	Four curve boundaries, smooth blending	Car hoods, panels
Bicubic Patch	Tangent and curvature continuity	Organic shapes, fairing
Bézier / B-spline Surface	Freeform 3D modeling	Car bodies, consumer goods

5. Applications in CAD and Industry

- **Product Design**: Automotive panels, consumer electronics, jewelry, conceptual forms.
- **Engineering Simulation**: Accurate geometric input for FEA, CFD, or kinematic analysis.
- **Manufacturing**: Mold/die design, tool path generation in CAM, 3D printing models.
- **Animation & Graphics**: Character modeling, special effects, digital content creation.

Mastery of curve and surface representations empowers designers and engineers to handle complex, aesthetically refined, and technically robust 3D models for diverse engineering and creative disciplines.